## Unified English Braille

## Guidelines for Technical Material

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## About this Document

This document has been produced by the Maths Focus Group, a subgroup of the UEB Rules Committee within the International Council on English Braille (ICEB). At the ICEB General Assembly in April 2008 it was agreed that the document should be released for use internationally, and that feedback should be gathered with a view to a producing a new edition prior to the 2012 General Assembly.

The purpose of this document is to give transcribers enough information and examples to produce Maths, Science and Computer notation in Unified English Braille.

This document is available in the following file formats: pdf, doc or brf. These files can be sourced through the ICEB representatives on your local Braille Authorities.

Please send feedback on this document to ICEB, again through the Braille Authority in your own country.

## Guidelines for Technical Material

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# Guidelines for Technical Material 

## 1 General Principles

### 1.1 Spacing

1.1.1 The layout of the print should be preserved as nearly as possible. However care should be taken in copying print spacing along a line as this is often simply a matter of printing style. Spacing should be used to reflect the structure of the mathematics. Spacing in print throughout a work is often inconsistent and it is not desirable in the braille transcription that this inconsistency should be preserved.
1.1.2 For each work, a decision must be made on the spacing of operation signs (such as plus and minus) and comparison signs (such as equals and less than). When presenting braille mathematics to younger children, include spaces before and after operation signs and before and after comparison signs. For older students who are tackling longer algebraic expressions there needs to be a balance between clarity and compactness. A good approach is to have the operation signs unspaced on both sides but still include a space before and after comparison signs. This is the approach used in most of the examples in this document.
1.1.3 There are also situations where it is preferable to unspace a comparison sign. One is when unspacing the sign would avoid dividing a complex expression between lines in a complicated mathematical argument. Another is when the comparison sign is not on the base line (for example sigma notation where i equals 1 is in a small font directly below).
1.1.4 When isolated calculations appear in a literary text, the print spacing can be followed.

### 1.2 Underlying rules for numbers and letters

Listed below is a summary of the rules for Grade 1 mode and Numeric mode as they apply to the brailling of numbers and letters in mathematics. Refer to the complete versions of these rules for more detail.

### 1.2.1 Grade 1 mode

A braille symbol may have both a grade 1 meaning and a contraction (i.e. grade 2) meaning. Some symbols may also have a numeric meaning. A grade 1 indicator is used to set grade 1 mode when the grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.

Note that if a single letter (excluding a, i and o) occurs in an algebraic expression, it can be misread as a contraction if it is "standing alone" so may need a grade 1 indicator. The same is true of a sequence of letters in braille that could represent a shortform, such as ab or ac, if it is "standing alone".

A letter, or unbroken sequence of letters is "standing alone" if the symbols before and after the letter or sequence are spaces, hyphens, dashes, or any combination, or if on both sides the only intervening symbols between the letter or sequence and the space, hyphen or dash are common literary punctuation or indicator symbols. See the General Rule for a full definition of "standing alone".

### 1.2.2 Numeric mode

Numeric mode is initiated by the "number sign" (dots 3456) followed by one of the ten digits, the comma or the decimal point.

The following symbols may occur in numeric mode: the ten digits; full stop; comma; the numeric space (dot 5 when immediately followed by a digit); simple numeric fraction line; and the line continuation indicator. A space or any symbol not listed here terminates numeric mode, for example the hyphen or the dash.

A numeric mode indicator also sets grade 1 mode. Grade 1 mode, when initiated by numeric mode, is terminated by a space, hyphen or dash. Therefore while grade 1 mode is in effect, a grade 1 indicator is not required except for any one of the lowercase letters a-j immediately following a digit, a full stop or a comma. (Note that Grade 1 mode, when initiated by numeric mode, is not terminated by the minus sign, : : : ..)

### 1.3 Print Symbols

One of the underlying design features of UEB is that each print symbol should have one and only one braille equivalent. For example the vertical bar is used in print to represent absolute value, conditional probability and the words "such that", to give just three examples. The same braille symbol should be used in all these cases, and any rules for the use of the symbol in braille are independent of the subject area. If a print symbol is not defined in UEB, it can be represented either using one of the seven transcriber defined print symbols in Section 11, or by using the transcriber defined shape symbols in Section 14.

### 1.4 Format

```
:%:: continuation indicator
```

1.4.1 In print, mathematical expressions are sometimes embedded in the text and sometimes set apart. When an expression is set apart, the braille format should indicate this by suitable indentation, for example cells 3 with overruns in 5 or cell 5 with overruns in 7 . An embedded expression which does not fit on the current braille line should only be divided if there is an obvious dividing point. Often it is better to move the whole expression to the next braille line.
1.4.2 When dividing a mathematical expression, choice of a runover site should follow mathematical structure:

- before comparison signs
- before operation signs (unless they are within one of the mathematical units below)
- before a mathematical unit such as
o fractions (and within the fraction consider the numerator and denominator as units)
o functions
o radicals
o items with modifiers such as superscripts or bars
o shapes or arrows
o anything enclosed in print or braille grouping symbols
o a number and its abbreviation or coordinates
Usually the best place to break is before a comparison sign or an operation sign. Breaking between braille pages should be avoided.
1.4.3 When an expression will not fit on one braille line and has to be divided, the use of indentation as suggested in 1.4.1 should make it clear that the overrun is part of the same expression. However in the unlikely case where the two portions could be read as two separate expressions the continuation indicator (dot 5 ) should be placed immediately after the last cell of the initial line.
$(a+b+c+d+e)(f+g+h+i+j)=(1+2+3+4+5)(6+7+8+9+10)=600$



### 1.5 Typeforms

In mathematics, algebraic letters are frequently italicised as a distinction from ordinary text. It is generally not necessary to indicate this in braille. However, when bold or other typeface is used to distinguish different types of mathematical letters or signs from ordinary algebraic letters, e.g. for vectors or matrices, this distinction should be retained in braille by using the appropriate typeform indicator. See Section 2.7 for the emphasis of individual digits within numbers.

### 1.6 Capitalisation

In mathematics and science, strings of capital letters often occur, for example in a geometrical name, in a physics formula or in genetics. Such strings should always be uncontracted. Capital word indicators (double caps) are normally used. See Section 16 for advice on capital letters in chemical formulae. It is preferable to also use this approach in genetics or other topics where there are frequent changes of case within a sequence of letters.
rectangle $A B C D$


### 1.7 Use of Grade 1 indicators

| : | grade 1 symbol indicator |
| :---: | :---: |
| : : | grade 1 word indicator |
| : : : : | grade 1 passage indicator |
| : | grade 1 passage terminator |
| : $:$ : : $:$ | : :: grade 1 passage indicator on a line of its own |
| :: : : : : : | $: \vdots \quad$ grade 1 passage terminator on a line of its own |

1.7.1 Grade 1 indicators will not be needed for simple arithmetic problems involving numbers, operation signs, numerical fractions and mixed numbers.

Evaluate the following:

$$
3-21 / 2=
$$


1.7.2 Simple algebraic equations which include letters but no fraction or superscript indicators may need grade 1 symbol indicators where letters stand alone or follow numbers. (See Section 1.2 for the underlying rules and Section 3.2 for more examples)

$$
\begin{array}{lll}
y=x+4 c \\
\vdots: O: ~: ~: ~: ~: ~: ~: ~: ~: ~: ~: ~: ~: ~: ~
\end{array}
$$

1.7.3 More complex algebraic equations are best enclosed in grade 1 passage indicators. This will ensure that isolated letters and indicators such as superscript, subscript, fractions, radicals, arrows and shapes are well defined without the need for grade 1 symbol indicators.

Consider the following equation:

$$
3 x-4 y+y^{2}=x^{2}
$$



Note that this particular equation could also be written
because the left hand side of the equation is in grade 1 mode following the numeric indicator (see Section 1.2).

Similarly
$\frac{x^{2}+2 x}{1+x^{2}}=1$
(fraction: $x$ squared plus $2 x$ all over $1+x$ squared close fraction) can be safely written as

but could also be written


See Section 11.5 for more examples of the use of grade 1 passage indicators.
1.7.4 If a complex algebraic expression does not include a comparison sign (such as an equals sign) then it is unlikely to include interior spaces in braille (see Section 1.1.2). In this case a grade 1 word indicator will be enough to ensure that superscript, subscript, fractions, radicals, arrows and shape indicators are well defined without the need for grade 1 symbol indicators.

Evaluate $\sqrt{\left(y-x^{2}\right)}$


See Section 7.3 for more examples of the use of grade 1 word indicators.
1.7.5 When entire worked examples or sets of exercises are enclosed in grade 1 passage indicators, the grade 1 indicators can be preceded by the "use indicator" and placed on a line of their own.

Solve the following quadratic equations:

1. $x^{2}-x-2=0$
2. $x^{2}-4 x-3=0$
3. $2 x^{2}-x=1$

1.7.6 When only a few contracted words are involved, the grade 1 passage indicator can be used to enclose entire worked examples and sets of exercises. In this situation any words occurring in the exercises will be written in uncontracted braille and isolated letters will not need letter signs. Where there is more text involved it is better to stay in grade 2 and use grade 1 passage, word or symbol indicators only as required.
1.7.7 In the examples in this document, grade 2 mode is assumed to be in effect, and grade 1 indicators have been included according to the guidelines in this section. Minimising the number of indicators must be balanced against reducing clutter within the expression itself. A grade 1 symbol indicator which occurs half way through an expression may be more disruptive to the reader than a word or passage indicator, even if these take up more cells. It is also important to use a consistent approach when transcribing a particular text. Overall the focus should be on mathematical clarity for the reader.

Further guidance will be given when more feedback has been received from students.

## 2 Numbers and Abbreviations

Refer to Section 1.2 for a summary of the rules for Grade 1 mode and Numeric mode as they apply to the brailling of numbers and letters in mathematics.

The braille representation of numbers such as dates and times should reflect the punctuation used in print.

### 2.1 Whole numbers

```
4 5 6
::0:!:0:0
3,000
:%:0:0:0:0:0:0:0
5000000
:!:
```

Calling seat numbers 30-59.


In the 60's


In the 60s


In the '60s


Phone 09-537 0891


For negative numbers see 4.2.

### 2.2 Decimals

### 8.93

$\because \because \because \bullet \bullet \bullet!$
0.7
: : : : : :
. 7
: : : :

Is the number in the range 2-5.5?

.8 is a decimal fraction.


For recurring decimals see Section 12 (bars, dots etc. over and under)

### 2.3 Dates

28-5-2001


5-28-01
: : : : : : : : : : : : : : : : : : : : :
2001/5/28
: : : : : : : : : : : : : : : : : : : : : : : : : : :
2001.5.28
: : : : : : : : : : : : : : : : : : :
28/5-31/5


### 2.4 Time

```
5:30 pm
```


5.30
: :
08.00
: : : : : : : : : : :
1300
:
6-7 a.m.
: : : : : : : : : : ! : : : : : :
6:15-7:45
: : : : : : : : : : : : : : : : : : : : : : : : :

### 2.5 Ordinal numbers

```
1st
:!:!:!:
```

2nd or 2d
: : : : : : : : : : : : : : : : : : : : :
3rd or 3d
: : : : : : : : : : : : : : :
4th
: : : : : : : :
1 er
: : : : : : :

### 2.6 Roman Numerals

Roman numerals should be brailled as if they were normal letters using the rules for grade 1 mode. Note that " $v$ " and " $x$ " will have grade 1 indicators but " $i$ " will not.

Read parts I, II and V.


Answer questions i , vi and x .


CD


### 2.7 Emphasis of Digits

If a typeform indicator applies to a digit or digits within a number, the numeric indicator needs repeating after any typeform indicator. If the first digit is affected then the typeform indicator should be placed before the numeric indicator.


For recurring decimals see Section 12 (bars, dots etc. over and under)

### 2.8 Ancient Numeration systems

Braille symbols to represent numerals from other number systems may be devised for each situation using transcriber defined print symbols. These should be defined either on the special symbols page or in a transcriber's note. (See example in Section 11.6.)

### 2.9 Hexadecimal numbers

Hexadecimal numbers occur in a computer setting and are made up of the digits 0 to 9 and the letters A to F. They should be treated the same as any other string of letters and numbers.

Fatal exception 0E has occurred at 0028:C00082CD


### 2.10 Abbreviations

The following signs are used for special print symbols:


Note that the Rand (South Africa) is written in print as a normal capital R so would be brailled as such.

Note that the foot or minute may be shown in print by an apostrophe (') and the minute or second by a non directional double quote ("). This usage can be followed in braille.

Follow print for order, spacing, capitalisation and punctuation of abbreviations. (If it is unclear in print whether there is a space between a number and its unit, or if print spacing is inconsistent, then it is recommended that a space is inserted in the braille.)

Where should I write the dollar sign, US $\$$ or $\$$ US?


30 cents can be written as $\$ 0.30$, 30c or $30 \phi$.


In South Africa, this would cost R13.51.


Before decimalisation, $£ 1.75$ was $£ 115$ s so half of it was 17 s 6 d or $17 / 6$.


Half a yard is 1 ft 6 in or $1^{\prime} 6^{\prime \prime}$ which is about 45 cm or 0.45 m .


1 L of water weighs 1000 g which is about 2 lbs 4 oz .


Is the speed limit 30 mph or $50 \mathrm{~km} / \mathrm{h}$ ?
!:!:0:!:0:0:0
!:!:0:!:0:0:0

Water freezes at $0^{\circ} \mathrm{C}$ or $32^{\circ} \mathrm{F}$.


To decrease by $15 \%$ multiply by 0.85 .

$$
\begin{aligned}
& \text { : : : : : : : : : : }
\end{aligned}
$$

Add 1 can of beans, 1 c of flour, 2 T of oil and 1 tsp of baking powder.

There are $360^{\circ}$ in a revolution, $60^{\prime}$ in a degree and $60^{\prime \prime}$ in a minute.

One complete orbit lasts 2 yr 5 m 15 d 7 h 17 min and 45s.


A 6 V battery will cause a current of 3 A to flow through a resistance of $2 \Omega$.


The reading was 15 mHz .

The pattern says k4 p1 s11 k1 psso.

$$
\begin{aligned}
& \text { : : : : : : : : : } \\
& 1 \AA=\frac{1}{10,000} \mu
\end{aligned}
$$

## 3 Signs of operation, comparison and omission

Operation signs:

| : $:$ : $:$ : $:$ | $+$ | plus |
| :---: | :---: | :---: |
| : $: \square$ | - | minus (when distinguished from hyphen) |
|  | x | times (multiplication cross) |
|  | $\div$ | divided by (horizontal line between dots) |
| : : : : : | $\pm$ | plus or minus (plus over minus) |
| : : : : : | $\mp$ | minus or plus (minus over plus) |
| : $:$ : : : : | - | multiplication dot |

Comparison signs:
: : : : : : : = equals
$\because::!$ < less than, or opening angle bracket

$::::$ : : : $:$ : less than or equal to
$::: 0:!\geq$ greater than or equal to
: : : : : : : : : : : $\neq$ not equal to (line through an equals sign)
$\vdots:: \vdots: \quad \simeq \quad$ approximately equal to (tilde over horizontal line)
$\vdots:: \quad$ : $: \quad \approx \quad$ approximately equal to (tilde over tilde)

Less common signs of comparison:
$\because:!:!\quad \because \quad$ is much less than

$: 0: 0: 0 \quad \cong$ tilde over equals sign (approximately equal)
$:: 0: \vdots \quad \doteqdot \quad$ equals sign dotted above and below (approximately equal)
$\because:: 0:: \quad \Omega \quad$ equals sign with bump in top bar (difference between or approximately equal)
$\vdots::: \quad \equiv: \quad \equiv \quad$ equivalent to (three horizontal lines)

Ratio
$\vdots:: \quad: \quad$ ratio sign (represented by a colon as in print)
(see also Section 11 for signs of operation and comparison used in set theory, group theory and logic)

### 3.1 Examples

In most of the examples below, operation signs are unspaced from preceding and following terms but comparison signs are spaced. The first two examples show the use of extra space for the younger learner. Follow the guidelines in Spacing (Section 1.1.2).

```
\(3+5=8\)
```



```
\(8-5=3\)
\(\vdots:!\quad \vdots: \vdots \quad \vdots!: \vdots\)
\(3 \times 5=5 \times 3=15\)
: : : : : : : : : : : : : : : : : : : : : : : : : : : : : : :
\(2 \mathrm{~cm}+4 \mathrm{~cm}=6 \mathrm{~cm}\)
```


$200 \mathrm{~g} \times 5=1 \mathrm{~kg}$
: : : : : : : : : : : : : : : : : : : ! : : : : : : : : : : :
$5.72 \mathrm{~m} \div 10=57.2 \mathrm{~cm}$

$15 \pm 0.5$
: : : : : : : : : : : : : : : : :
Area $=$ bh $=5 \cdot 3=15$

$3.9 \times 4.1 \simeq 16$
: : : : : : : : : : : : : : : : : : : : : : : : : : : : : :
$5-3 \neq 3-5$
: : : : : : : : : : : : : : : : : : : : : : :

A scale of 1:200
: : : : : : : : : : : : : : :
(although the ratio sign is used to compare two numbers, it is best treated as an operation sign for purposes of spacing)
$2: 4=6: 12$


### 3.2 Algebraic Examples

In the algebraic examples below note the use of the grade 1 indicator whenever a letter is "standing alone" according to the definition in Section 1.2 and so could read as a contraction. In the last example, the colon itself could read as the cc contraction.

If $\mathrm{y} \propto \mathrm{x}$ then $\mathrm{y}=\mathrm{kx}$


Find $\theta$ if $0 \leq \theta \leq 2 \pi$

| $y=x+4$ |
| :---: |
|  |
| $2 \mathrm{y}=2 \mathrm{c}-4$ |
| : : : : : : |
| $d+a b=a c$ |
|  |
| The ratio $\mathrm{x}: \mathrm{y}$ |

### 3.3 Use of the braille hyphen

If the minus sign and hyphen are indistinguishable in print then the braille hyphen can be used for both. However most maths and science texts show the minus sign as slightly longer than a hyphen in print so the dot 5 form is best used throughout.
interest-rate - inflation-rate


The temperature was 15-17. (ambiguous print)


### 3.4 Positive and negative numbers

Sometimes positive and negative numbers, as opposed to added or subtracted numbers, are shown in print by a plus sign or a minus sign being written as a left superscript (that is being written above and to the left of the number). This can be shown in braille using the superscript indicator. (See Section 7)

Graduate the $x$ axis from -4 to +5 .
(with the plus and minus signs in the central position in print)


Evaluate ${ }^{-} 2+{ }^{-} 3$
(with the minus signs in the superscript position in print)


### 3.5 Calculator keys

When numbers and operators are enclosed in boxes to show that calculator keys are to be pressed, it may not be helpful to use the shape enclosure indicators in Section 14.3. If the boxes need to be shown then an alternative approach is to use the underline indicator and a transcriber's note.


### 3.6 Omission marks in mathematical expressions

```
::::!:! _ _ long dash
: :: :!:! _ underscore
:%:% visible space
:!: :!:: : : \square \square square (see Section 14)
3+7= (omission shown as a centred horizontal line in print)
```



```
3+7 = __ (omission shown as a low horizontal line in print)
```



```
3+7 = ? (omission shown as a question mark in print)
```



```
3\square7=10 (omission shown as an empty square in print)
```



```
3 7 = 10 (omission shown as an unmarked gap in print)
```



```
\frac{9}{12}=\frac{3}{-}\quad\mathrm{ (omission shown within a fraction in print)}
```



Note that the visible space is the same symbol as the closing root indicator (defined in section 12). If the radicand is to be filled in then the symbol would be repeated to terminate the root.

```
5 = \sqrt{}{}
```



## 4 Spatial Layout and Diagrams

### 4.1 Spatial calculations



Where horizontal lines are needed within children's sums, horizontal line mode should be used. The layout of the calculation can follow print, though feedback from teachers working with students should also be taken into account. These guidelines can also be applied for other spatial arrangements such as financial statements or accountancy texts.

Columns to be added should not contain numeric indicators or operation signs. This can be achieved by aligning numeric indicators vertically - a numeric indicator followed by a space still initiates numeric mode.

Alternatively use the numeric passage indicator and the numeric terminator which set numeric mode and grade 1 mode for the enclosed text. In a numeric passage numeric indicators are not used, and any lowercase letter a to $j$ is preceded by a grade 1 indicator.

The line above and below spatial calculations should either be blank, or should only contain the numeric passage indicator or terminator.

### 4.1.1 Addition or subtraction

456
$+34$
490


The second version below illustrates the use of the numeric passage indicator. The placement of the operation sign does not need to follow print, but can be adjusted to suit local teaching practices. Indentation has been used to keep the columns of the calculation clear of numeric indicators.


### 4.1.2 Long multiplication

123
$\times 12$
246
123
1476


### 4.1.3 Division

The spaced vertical line segment (dots 456) can be used to represent the curved or straight line used in print to denote " 5 into 15 ". A single space may also be acceptable. The layout of division calculations can be adjusted to suit local teaching practices.
$5 \longdiv { 1 5 } = 3$


Below, the version on the left follows print layout, while the version on the right is an Australian example which has been adjusted to make it easier for the student to replicate.


### 4.1.4 Spatial fractions for teaching purposes



### 4.1.5 Carryovers

Insert spaces between the digits in braille so that the small numbers can be placed on their own row above or below the central calculation. The layout of this type of example can be adjusted to suit local teaching practices.
$3^{1} 2$
$\frac{1}{1} 1 \frac{4}{8}$
::


### 4.1.6 Cancellation

$\because:!:!$ line through previous item (defined in Chapter 12)
When cancellation of numbers or expressions is shown in print, it is sometimes better to preserve the usual braille form and explain the process in a transcriber's note. Take the age of the student into account and check with local teaching practices.

For the fraction example below, two possible approaches are shown.

$$
\frac{{ }^{1}}{5} \times \frac{8}{و_{3}}=\frac{8}{15}
$$

Spatial arrangement

```
:: :!
```


: : :

Using cancellation lines within general fraction indicators (See Chapter 6)


### 4.1.7 Numbered calculations

If a series of calculations are numbered, then numeric passage indicators can be used to enclose the whole series. However it is recommended that the numeric indicator is also used before each item number. Ex:


### 4.2 Tally marks

::: :
: : : : : : : : : : : :
tally mark (vertical line segment)
4 tally marks with strike through (representing 5 items)

The horizontal or diagonal strike-through represents the counting of a fifth item. This can either be shown as a fifth tally mark in braille or the " 4 tally marks with strike-though" symbol can be used. Ex:

Eighteen HH HH HH III


### 4.3 Tables

Horizontal line mode is used to underline the column headings. Two spaces are left between columns. If numbers are to be added they should be right adjusted. Ex:
$y=x^{2}-3$

| $x$ | $y$ |
| :--- | :--- |
| -1 | -2 |
| 0 | -3 |
| 1 | -2 |
| 2 | 1 |
| 3 | 6 |



It is also acceptable to include a boxing line above and below the table.

### 4.4 Diagrams

This section does not attempt to give advice on the design of tactile diagrams or on the range of production methods available.

### 4.4.1 Horizontal Line Mode

Most transcribers have experimented with the use of braille symbols for drawing simple box shapes or family trees. Although these are often not as readable as a raised line drawing, they still have their uses.

If an unbroken braille line begins with the horizontal line mode indicator:: : $:$ then all the other cells in that line will be read as shapes rather than symbols.

If vertical lines are also shown using the vertical line segment : then a back translation process or a tuned in reader can be clear that this is a diagram rather than a string of text.


Refer to the Line Drawing Rule for a list of useful cell patterns to use within horizontal line mode. Special mention is made of arrow symbols which can be used without terminating horizontal line mode. The following two examples illustrate the use of arrow symbols and also include unlisted dot patterns for the shapes of the hollow and solid circles used in print to represent the open or closed end of an interval.

In the print example below a number line is graduated from -2 to 3 and an interval is marked above it starting at 0 and finishing with an open circle at 2 . In the braille version, UEB arrow mode is used for the left and right pointing arrows (See Section 13) and horizontal line mode is used between them. The interval itself is shown as a line finishing with the dot pattern $\vdots:$ for the empty circle.


In the second example below the interval starts with a solid circle at -2 and finishes at 0 . In the braille version, the dot pattern $\vdots:$ has been used for the solid circle.


Note that in this second braille example, the horizontal line mode indicator: : : could not precede the circle symbol without obscuring the position of that start point on the number line. In any diagram, tactual clarity may be deemed more important than technical ambiguity.

Always bear in mind that a well executed raised line drawing will generally be more readable than a diagram created using braille cell patterns.

See Section 16 for examples of chemistry structures drawn using horizontal line mode and vertical and diagonal line segments.

### 4.4.2 Labelling diagrams

This section does not attempt to give advice on how to effectively label a diagram in braille so that it is clear to the reader which part of the diagram each label refers to. Rather, it lists code accommodations which can be made within the content of certain labels when space is at a premium.

- When labelling points, lines etc with single letters, these can be consistently shown as two cells by omitting the grade 1 indicator for all capital letters and including it for all lowercase letters including a, i and o.
- When graduating an axis with negative numbers, the hyphen could be used instead of the minus sign (ie the dot 5 prefix could be omitted). Include a transcriber's note.
- When writing coordinates such as $(2,-3)$, any of the following could be considered: omitting the space; omitting the dot 5 prefix from the round brackets; omitting the dot 5 prefix from the minus sign. Include a transcriber's note.
- When labelling a line or curve with its equation, the space either side of the equals sign could be omitted.
- When labelling angles, the degree sign could be omitted. Include a transcriber's note.


## 5 Grouping Devices (Brackets)



Print brackets are usually unspaced from the items they enclose and the same should be done in braille. See Section 15 for the layout of matrices and vectors.

Let the midpoint of $A(3,-5)$ and $B(0,4)$ be the point ( $x, y$ )

$[2(x+y)] \div 4<10$

$|-6|=|6|=6$ (absolute value)


Consider the sequence $<T_{n}>$.


## 6 Fractions

$\vdots: \vdots \quad$ simple numeric fraction line
$\vdots:: \vdots: \quad$ general fraction line
: :: :: general fraction open indicator
: :: : $\quad$ general fraction close indicator

### 6.1 Simple numeric fractions

A simple numeric fraction is one whose numerator and denominator contain only digits, decimal points, commas or separator spaces and whose fraction line in print is drawn between the two vertically (or nearly vertically) arranged numbers. In such a case a numeric fraction line symbol may be used between the numerator and denominator and continues the numeric mode.
$\frac{5}{8}$ of the class are boys.


Calculate $\frac{5.7}{2,000}$


### 6.2 Mixed numbers

Mixed numbers should be treated as two unspaced numeric items.
$21 / 2$ cups of sugar

$1750 \mathrm{~cm}=13 / 4 \mathrm{~m}$


### 6.3 Fractions written in linear form in print

The numeric fraction line need not be used when the print is expressed linearly using an ordinary slash (oblique stroke) symbol. In such a case the same symbols can be used as in print.

3/8 of the class are girls.


### 6.4 General fraction indicators

If the numerator or denominator is not entirely numeric as defined in 6.1, then the general fraction indicators should be used. After the opening indicator the numerator expression is written, then the general fraction line symbol, then the denominator expression and finally the closing indicator. Both numerator and denominator may be any kind of expression whatever, including fractions of either simple numeric or general type.

In the verbalisations below, quotation marks have been used to reflect the spatial arrangement used in print.

```
\(y=\frac{x}{2} \quad(y=-x\) over \(2 ")\)
```



```
\(\frac{21 / 2}{x+y} \quad\) (two and a half over " \(\mathrm{x}+\mathrm{y}\) ")
```



```
\(\frac{2 / 3}{5} \quad(2 / 3\) all over 5\()\)
: : : : : : : : : : : : : : : : : : : : : : : : : : :
\(\frac{\frac{x}{2}+\frac{y}{3}}{x+y} \quad\) (fraction "x over 2" + "y over 3" all over "x+y")
```


speed $=\frac{\text { distance }}{\text { time }}$
(speed = distance over time, showing alternative useage of grade 1 indicators)




### 6.5 Extra Examples

These examples are included to further illustrate the difference in print between simple numeric fractions as defined in 6.1, and fractions which will need general fraction indicators in braille as defined in 6.3.

Simple numeric fractions
$\frac{3}{4} \quad \frac{73}{4} \quad \frac{3}{4.2} \quad 5 \frac{3.3}{4,200} \quad 6 \frac{3}{10000}$

Fractions needing general fraction indicators because they contain letters

| $\frac{x}{y}$ | $\frac{x}{4}$ | $\frac{3}{b}$ | $\frac{a}{b+c}$ | $\frac{3 x}{4}$ | $\frac{x}{4,000}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |

Fractions needing general fraction indicators because they contain something more than digits, commas or decimal points.
$\frac{3^{2}}{6}$
$\frac{\$ 55}{5}$
$\frac{24 m}{3 c m}$
$\frac{?}{4}$
$\frac{2 / 3}{4}$
$\frac{2-5}{3}$

## 7 Superscripts and subscripts

$\vdots: \vdots$ level change down (subscript)
$: \vdots: \quad$ level change up (superscript or exponent)
$\vdots: \vdots: \%$ expression directly below
:: : : : $0 \quad$ expression directly above
$\vdots: \vdots \quad$ braille grouping open
: :: : braille grouping close

### 7.1 Definition of an item

The scope of any of the four level change indicators, that is, the symbol(s) affected by it, is the next "item". An item is defined as any of the following groupings if immediately after the level change indicator:

1. An entire number, i.e. the initiating numeric symbol and all succeeding symbols within the numeric mode thus established (which would include any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
2. An entire general fraction, enclosed in fraction indicators (see Section 6).
3. An entire radical expression, enclosed in radical indicators (see Section 8).
4. An arrow (see Section 13).
5. An arbitrary shape (see Section 14).
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces (see Section 5).
7. Any expression enclosed in the braille grouping indicators.
8. If none of the foregoing apply, the item is simply the next individual symbol.

### 7.2 Superscripts and subscripts within literary text

Note that if a superscript or subscript appears within a grade 2 passage, it will need a grade 1 indicator.

The area is $6 \mathrm{~m}^{2} \quad$ (The area is 6 m squared)


The points $P_{1}$ and $P_{2} \quad$ (The points $P$ sub 1 and $P$ sub 2)


Smith wrote a paper ${ }^{56}$ which says . . . (super 56 indicating a footnote)


The formula for water is $\mathrm{H}_{2} \mathrm{O} \quad$ (The formula for water is H sub 2 endsub O )


### 7.3 Algebraic expressions involving superscripts

Refer to the definition of an item in 7.1 to decide when braille grouping symbols are required. Note that the braille grouping symbols themselves have an alternative grade 2 meaning.

In the verbalisations below, quotation marks have been used to reflect the spatial arrangement used in print.

$$
\begin{aligned}
& x^{2} \quad \text { ( } x \text { squared) } \\
& \text { : : : : : : : } \\
& x^{2} y \quad(x \text { squared times } y) \\
& \text { :! : : : : : : : : : } \\
& x^{2 y} \quad(x \text { to the } 2 y) \\
& \text { : : : : : : : : : : : : : : : : : : }
\end{aligned}
$$

```
\(x^{y}+1 \quad(" x\) to the \(y "+1)\)
:
\(x^{y+1} \quad(x\) to the \(" y+1 ")\)
    : : : : : : : : : : : : : : : : : : : : : :
\(x^{y+1}+3 \quad(x\) to the \(" y+1 "+3)\)
: : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : :
\(x^{2 / 3} \quad\) ( \(x\) to the two thirds)
:
\(\frac{x^{2}}{3} \quad\) (x squared over 3)
: : : : : : : : : : : : : : : : : : : : : : : : :
\(x^{1 / 2 y} \quad(x\) to the "half \(y ")\)
: : : : : : : : : : : : : : : : : : : : : :
\(x^{1 / 2} y \quad(" x\) to the half" \(y)\)
:
\(x^{\frac{a}{b}} y=x \quad(x\) superscript "a over b " end superscript \(\mathrm{y}=\mathrm{x}\) )
```



```
If \(\frac{x^{2}+2 x}{1+x^{2}}=1\)
\(x^{2}+2 x=1+x^{2}\)
\(x=1 / 2\)
        (If fraction "x squared \(+2 x\) " all over "1 \(+x\) squared" end fraction \(=1\)
        \(x\) squared \(+2 x=1+x\) squared
        \(x=1\) over 2 )
```




```
\(\because: \quad \vdots: \quad \vdots \quad \vdots:!\)
```


### 7.4 Multiple levels

Note that a superscript which itself has a superscript does not fit the above definition of an item. In such cases braille grouping symbols are required.
$e^{x}$ (e to the $x$ squared)
: : : : : : : : : : : : : : : : : : : : :
$e^{\left(x^{2}\right)}$
(e to the open parenthesis $x$ squared close parenthesis)
: : : : : : : : : : : : : : : : : : : : : :
$P_{x}$
$P_{x_{i}} \quad$ (P with an $x$ sub $i$ in the subscript position)


### 7.5 Negative superscripts

Negative superscripts must be enclosed in braille grouping symbols. (This is because a minus sign can be an item in its own right, as in 7.6 below)

```
\(0.0045=4.5 \times 10^{-3} \quad(0.0045=4.5\) times 10 to the minus 3\()\)
```



```
\(v=60 \mathrm{~ms}^{-1} \quad(\mathrm{v}=60 \mathrm{~ms}\) to the minus 1\()\)
```



```
\(\mathrm{a}^{-2 \mathrm{~b}} \quad\) (a to the minus 2 b power)
: : : : : : : : : : : : : : : : : : : : : : :
```


### 7.6 Examples from Chemistry

See Section 16 for more examples.
$\mathrm{CH}_{4}+2 \mathrm{Cl}=\mathrm{CH}_{3} \mathrm{Cl}+\mathrm{HCl}$


Ions $\mathrm{H}^{+}, \mathrm{Cl}^{-}$and $\mathrm{Ca}^{2+}$


### 7.7 Simultaneous superscripts and subscripts

If more than one superscript or subscript apply, work from bottom to top, or left to right. If the print indicates by the placing of the subscript that it is being applied after the superscript then the order can be reversed.
$x_{1}^{2}=y_{2}^{3} \quad(x$ sub 1 squared equals y sub 2 cubed)

$X^{2} k \quad(x$ squared sub $k)$
: : : : : : : : : : : : : : :

### 7.8 Left-displaced superscripts or subscripts

Sometimes in print a superscript or subscript is written to the left of the base symbol instead of to the right. These are handled simply by using the corresponding ordinary index expression prior to the base symbol.
${ }_{92}^{238} U$ (U with 92 written below left and 238 written above left)

$-2+-3=-5 \quad$ (minus $2+$ minus $3=$ minus 5 with minus signs in the superscript position)

### 7.9 Modifiers directly above or below

If something is written directly above or below a term rather than to the right or left, use the directly above indicator or directly below indicator instead of the superscript or subscript indicator.

Common modifers such as the bar, arrow, dot, tilde, hat or arc are treated separately in Section 12.
$\sum_{x=1}^{n} x_{i}{ }^{2}$
(The sum from $\mathrm{i}=1$ to n of x sub i squared with summation limits directly below and above a capital sigma)

$\lim _{x \rightarrow a} f(x)=1$
(The limit, as $x$ tends to a, of $f$ of $x,=1$, with $x$ arrow a directly below lim)

$\bar{x} \quad$ (x with bar over it)
$\because:!\quad$ (see Section 12.1)

## 8 Square Roots and other radicals

$\because::$ : open radical (root)
$\because:: \quad$ : close radical (root)
$\because: \vdots$ : $: \vdots$ square root sign without vinculum

### 8.1 Square roots

The expression inside the square root sign in print (the radicand) should be preceded by the open radical sign and followed by the close radical sign. The radicand itself may be any expression whatsoever, and may therefore contain radicals as well as other mathematical structures. Note that both the opening and closing radical signs have alternative grade 2 meanings so may need grade 1 indicators.

```
\sqrt{}{9}=3 (the square root of 9=3)
::0:!:!:!: !: :% 足:!
```

(Note that the square root of a simple number still needs to be terminated by the close radical sign.)

```
r=\sqrt{}{\mp@subsup{x}{}{2}+\mp@subsup{y}{}{2}}\quad(r=\mathrm{ the square root of }x\mathrm{ squared + y squared end root)}
```


$\sqrt{\frac{783.2 \times 6.547}{0.4628}}$
(the square root of fraction 783.2 times 6.547 over 0.4628 end fraction end root) :
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
( $x=$ the fraction: minus b plus-or-minus the square root of $b$ squared minus 4ac end root all over $2 a$ )


### 8.2 Cube roots etc

In print the radical index, if present, is printed above and to the left of the radical sign. This index is placed in braille as a superscript expression immediately following the opening radical symbol.

$$
\begin{aligned}
& \sqrt[3]{8}=2 \quad(\text { the cube root of } 8=2) \\
& \text { : : : : : : : : : : : : : : : : : : ! : : : : ! : : : : } \\
& q=\sqrt[3]{x^{3}+y^{3}+z^{3}} \\
& \text { ( } q=\text { the cube root of } x \text { cubed }+y \text { cubed }+z \text { cubed end root) }
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt[m n]{x y} \quad \text { (the mn-th root of } x y \text { ) } \\
& \text { : : : : : : : : : : : : : : : : : : : : : : : } \\
& 81^{\frac{3}{4}}=(\sqrt[4]{81})^{3}=(\sqrt{\sqrt{81}})^{3}=(\sqrt{9})^{3}=3^{3}=27
\end{aligned}
$$

(81 to the three-quarters $=$ (the fourth root of 81 ) cubed $=$ (the square root of the square root of 81) cubed $=$ (the square root of 9 ) cubed $=$ three cubed $=27$ )


### 8.3 Square root sign on its own

Sometimes print omits the horizontal line (vinculum) above the radicand. In older books $\sqrt{ } 4$ could mean the square root of 4 , but in some modern contexts could be just a sequence of symbols.

When introducing simple roots to younger children, it is better to introduce the concept of opening and closing indicators from the start. If it is clear from the context that this is a square root, then it is best to assume an intended vinculum and to use the standard form. If, however, the square root sign is being used as an isolated graphic symbol then the dot 5 form can be used. Ex.
$\sqrt{4}=2$
$\sqrt{ } 4=2$
The $\sqrt{ }$ sign


## 9 Functions

If a function name is preceded or followed by a letter of the same font and alphabet, it may not be clear where the function name begins or ends. In print this is clarified using a variety of techniques. Take the example a times the cosine of t . In print this is often written acos $t$. Notice that here the a and $t$ are written in italics to show they are variables, and a small space is included either side of the function name. This technique is also used in print to distinguish word fragments other than functions.

The examples in this section are taken from secondary school science and mathematics but the same rules should be followed when dealing with functions within any subject area.

### 9.1 Spelling and capitalisation

Follow print for the spelling and capitalisation of function names.
Find the value of Cosine B.


### 9.2 Italics

Where letters before or after the function name are written in italics to indicate they are variables, the italics should be omitted in accordance with rule 1.3.

### 9.3 Spacing

Where the function name is preceded or followed by a letter, a space may be needed, to remove ambiguity as to where the function name begins and ends. The space is not needed if the function name is already separated by a bracket or by a braille indicator such as a capitalisation indicator, a Greek letter indicator or a fraction indicator. Care should be taken when a capital letter precedes a function name. This is summarised below.
9.3.1 If a function name is directly preceded or followed by a number, then the number should be written unspaced from the function name.

Sin 30

$3 \tan 45^{\circ} \quad(3 \tan 45$ degrees $)$

$4 \cos 5 x$
: : : : : : : : : : : : : :
9.3.2 Insert a space if a function name is followed directly by a lower case Latin letter with no intervening braille indicators or brackets.

```
logy
::%:: !::%
sin}0\quad(sin theta
:O:O:0:%
```

$\operatorname{Sec} A$

$\operatorname{Lim} \frac{x}{2} \quad(L i m$ "x over 2")

9.3.3 Insert a space if a function name is preceded directly by a lower or upper case Latin letter with no intervening braille indicators or brackets. Note that letters isolated by these extra spaces may need grade 1 indicators.
$x \sin 60$
: : : : :
$x \operatorname{Sin} 60$
:! : : : : : : : : : : : :
$X \log y$

$x \log y$

$\sin (A+B)=\sin A \cos B+\cos A \sin B$

$\sin 2 \beta=2 \sin \beta \cos \beta \quad(\sin 2$ beta $=2 \sin$ beta $\cos$ beta)


### 9.4 Trigonometric functions

Common trigonometric functions are Sine, Cosine, Tangent, Secant, Cosecant and Cotangent, usually abbreviated in print to sin, cos, tan, sec, cosec and cot.
Their inverses may be written in print as
$\sin ^{-1}$ ( $\sin$ superscript minus 1 ), $\cos ^{-1}, \tan ^{-1}, \sec ^{-1}, \operatorname{cosec}^{-1}$ and $\cot ^{-1}$.
or less commonly as arcsin, arccos, arctan, arcsec, arccosec and arccot.
You may also meet the associated hyperbolic functions
sinh, cosh, tanh, sech, cosech and coth and their inverses
If $\sin \theta=\frac{o}{h}=\frac{2}{4}$ then $\theta=\sin ^{-1} 0.5=30^{\circ}$
(If $\sin$ theta $=\mathrm{o}$ over $\mathrm{h}=2$ over 4 then theta $=$ sin to the minus 1 of $0.5=30$ degrees)


Prove $\sec ^{2} x=1+\tan ^{2} x \quad$ (Prove sec squared $x=1+\tan$ squared x )


Find the derivative of $\mathrm{f}(\mathrm{x})=\frac{\cosh x}{\sinh x^{2}}$
(Find the derivative of $f$ of $x=$ fraction $\cosh x$ over $\sinh x$ squared end fraction)



### 9.5 Logarithmic functions

The logarithmic function is usually written log or Log and may be followed by a subscript indicating the base. A logarithm to base e is called a natural log and is often abbreviated to In.

```
\(3 \log x\)
: : : : : : : : : : : : :
\(\log _{2} 8=3 \quad(\log\) base 2 of \(8=3)\)
: : : : : : : : : : : : : : : : : : : : : : : : : !
\(\ln e=1\)
```



```
\(\log a+\log b=\log a b\)
```



```
\(\int \tan x d x=\ln \cos x+c\)
(The integral of \(\tan x\) with respect to \(x=\) the natural \(\log\) of \(\cos x+a\) constant)
```


$\log _{b} x=\frac{\log _{a} x}{\log _{a} b}$
(The log base $b$ of $x=$ the log base $a$ of $x$ over the log base $a$ of $b$ )


### 9.6 The Limit function

Limit, lim, Im, It are all used to indicate limit, sometimes with capitals, sometimes without.

Note: see Section 13 for the representation of arrows.
$\operatorname{Lim}_{x \rightarrow a} f(x)=1$
(The limit, as $x$ tends to $a$, of $f$ of $x,=1$, with $x$ arrow a directly below lim )

$\operatorname{limit}_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1$
(The limit, as theta tends to 0 , of sin theta over theta, $=1$ )


### 9.7 Statistical functions

Probability is shown in print in many ways, including P , Prob or Pr. Other statistical functions include expectation which may be shown in print as E or Exp.
$\operatorname{Pr}(A$ and $B)=\operatorname{Pr} A+\operatorname{Pr} B$
(probability of $A$ and $B=$ probability of $A+$ probability of $B$ )

$\operatorname{Exp}(R)=\frac{n}{2}+1 \quad$ (The expectation of $R=$ "n over $\left.2 "+1\right)$


### 9.8 Complex numbers

Functions used in complex number theory include arg (argument), Re (real part), Im (imaginary part) and cis.
$\arg \left(z_{1} z_{2}\right)=\arg z_{1}+\arg z_{2}$
(arg of $z$ sub $1 z$ sub $2=\arg z \operatorname{sub} 1+\arg z \operatorname{sub} 2)$

```
:!:: : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : !
```


$z=r \operatorname{cis} \theta=r \cos \theta+i r \sin \theta$
$(z=r$ cis theta $=r \cos$ theta $+i r \sin$ theta $)$



## 10 Set Theory, Group Theory and Logic

| : : : : \% | $\cup$ | union (upright $U$ shape |
| :---: | :---: | :---: |
| : : : : | $\bigcirc$ | intersection (inverted U shape) |
| : : : : : | $\varnothing$ | null set (slashed zero) |
| : : : : | , | complement (prime sign) |
| : $:$ : : $:$ : 0 | $\epsilon$ | is an element of (variant epsilon) |
| : : : : : 0 | $\ni$ | contains as an element (reverse variant epsilon) |
| : : : : : | $\subset$ | contained in, is a subset of ( $U$ open to right) |
| :: : $:$ | $\bigcirc$ | contains, is a superset of (U open to left) |
| :: : : : | $\subseteq$ | contained in or equal to |
| :: : : : : | $\bigcirc$ | contains or equal to |
| : $:$ : $:$ | ᄃ | contained in, but not equal to (proper subset) |
| :: : : : : | $\supsetneq$ | contains, but is not equal to (proper superset) |


$\vdots::: \vdots: \vdots \unlhd$ is normal subgroup of or equal (closed "less than", line under)
 (closed "greater than", line under)
: :: : : : : : $\quad \exists$ normal subgroup but not equal (closed "less than", cancelled line under)
: : : : : : : inverse "normal subgroup but not equal" (closed "greater than", cancelled line under)

| : | $\checkmark$ | or (upright v shape) |
| :---: | :---: | :---: |
| : : : : : | $\wedge$ | and (inverted v shape) |
| : : : : : : | $\neg$ | "not" sign (line horizontal, then down at right) |
| : : : | $\vdash$ | assertion ("is a theorem" sign; "T" lying on left side) |
| : : : : : | $\dashv$ | reverse assertion ("T" lying on right side) |
| : : : : : | F | "is valid" sign (assertion with double stem on "T") |
| : : : : : | $=$ | reverse "is valid" sign |

If $A=\{1,2,3,4\}$
and $B=\{2,4,5,8\}$
is $3 \in A \cap B \quad$ (is 3 an element of $A$ intersection $B$ )
and is $A \cap B \subset A \cup B$ ? (and is $A$ intersection $B$ a subset of $A$ union $B$ )

$A^{\prime} \cup B^{\prime}=(A \cap B)^{\prime}$
(the union of $A$ complement and $B$ complement $=$ the complement of the intersection of $A$ and $B$ )


For the statements p and q
$[(p \vee q) \wedge \neg p]$ 卜 $q \quad$ (Either $p$ or $q$; and not $p$; therefore, $q$ )


## 11 Miscellaneous Symbols

| : | $\int$ | integral sign |
| :---: | :---: | :---: |
| : : : : : : | $\oint$ | closed line integral (small circle halfway up) |
| : : : : : : | $\partial$ | partial derivative (curly d) |
| : : : : : : | $\nabla$ | del, nabla (inverted capital delta) |
| : :\% : : | , | prime (when distinguished from apostrophe in print) |
| : : : : : : : : | $\propto$ | is proportional to (varies as) |
| : : : : : | $\sim$ | tilde (swung dash) |
| : : : $:$ : | $\wedge$ | caret (hat) |
| : : : : : : | * | asterisk |
| : : : : : | - | hollow dot |
| : : : : : : | । | vertical bar |
| : : : : : : | $\infty$ | infinity |
| : : : | ! | factorial sign (exclamation mark in print) |
| : : : : : | $\angle$ | angle sign |
| : : : : : : : | $\Varangle$ | measured angle sign |
| : : : : : : : : | L | measured right angle sign |
| : : : : : | \|| | parallel to |
| : : : : : | $\perp$ | perpendicular to |
|  | . | "therefore" (three dots in upright pyramid) |
| : : : : : | $\because$ | "since" (three dots in inverted pyramid) |
| : : : : | $\exists$ | "there exists" (reverse E) |
| : \%: : : | $\forall$ | "for all" (inverted A) |
| ¢ : : : : | @ | at sign |
| : : : : : : | 1 | backslash |
| : : : : : : | - | underscore |
| : : : : : : | \# | cross hatch |
| : : : $:$ | \& | ampersand |
| : : : | i | broken vertical bar |

```
:%::
\bullet:: :: !: 
\bullet!::! :! :!
:::::%
::::0:%
::%:%:
:::%:%:
first transcriber-defined print symbol
second transcriber-defined print symbol
third transcriber-defined print symbol
fourth transcriber-defined print symbol
fifth transcriber-defined print symbol
sixth transcriber-defined print symbol
seventh transcriber-defined print symbol
```


### 11.1 Spacing

In general, the spacing of symbols can follow print. However if a symbol is clearly being used as a sign of operation or comparison, follow the guidelines in Section 1.1.

### 11.2 Unusual Print symbols

If a print symbol is not defined in UEB, it can be represented either using one of the seven transcriber defined print symbols above, or by using the transcriber defined shape symbols in Section 14. (See example 11.5.9 below)

### 11.3 Grade 1 indicators

If the braille version of a print symbol also has a grade 2 meaning, and grade 1 mode is not already in force, then grade 1 indicators will be needed. Symbols in the list above for which this applies are the integral sign, the prime sign and the therefore sign.

### 11.4 Symbols which have more than one meaning in print

One of the underlying design features of UEB is that each print symbol should have one and only one braille equivalent. For example the vertical bar is used in print to represent absolute value, conditional probability and the words "such that", to give just three examples. The same braille symbol should be used in all these cases.

### 11.5 Examples

### 11.5.1

If $y=f(x)$ then the derivative is $\frac{d y}{d x}$ or $f^{\prime}(x)$ and the partial derivative is $\frac{\partial y}{\partial x}$.
(If $y=f(x)$ then the derivative is $d y$ over $d x$ or $f$ dash $x$ and the partial derivative is curly d y over curly $d x$ )


### 11.5.2

$$
\begin{aligned}
& \int_{2}^{3}(2 x+1) d x \\
& =\left[x^{2}+x\right]_{2}^{3} \\
& =\left(3^{2}+3\right)-\left(2^{2}+2\right) \\
& =12-6=6
\end{aligned}
$$

(the integral from 2 to 3 of $(2 x+1) d x=[x$ squared $+x]$ sub 2 super $3=(3$ squared +3 ) minus ( 2 squared +2 ) $=12-6=6$ )


Note: the spacing of the integral sign in print can be unclear or inconsistent. In braille it is best to have the integral sign unspaced from the function and treat its limits as subscripts and superscripts. The dx at the end means "integrate with respect to $x "$, and can also be written unspaced.

### 11.5.3

${ }^{n} C_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$
(super $n$ capital $C$ sub $r=$ enlarged brackets enclosing $n$ at the top and $r$ at the bottom $=$ fraction $n$ factorial over $r$ factorial times ( $n$ minus $r$ ) factorial end fraction)


Note: the binomial coefficient works better as a shape than a vector (refer to 14.3.3).

## 11.5 .4

* is distributive over ○ if

$$
a *(b \circ c)=(a * b) \circ(a * c)
$$

(asterisk is distributive over hollow dot if $a$ asterisk $(b$ hollow $\operatorname{dot} c)=(a$ asterisk $b)$ hollow dot $(a$ asterisk $c))$


Note: the hollow dot should not be used to represent the abbreviation for degrees, which is covered in Section 3.

### 11.5.5

If $f: X \rightarrow Y$ is a function then the relation $f^{-1}: Y \rightarrow X$ is itself a function if and only if $\forall y \in Y \exists x \in X$ such that $f(x)=y$
(If $f$ from $X$ to $Y$ is a function then the relation $f$ super minus 1 from $Y$ to $X$ is itself a function if and only if for all $y$ in $Y$ there exists $x$ in $X$ such that $f$ of $x=y$ )


Note: see Section 13 for the representation of arrows.

### 11.5.6

$\{(x, y) \mid x+y=6\}$
(The set of $(x, y)$ such that $x+y=6$ )


### 11.5.7

Babylonian numerals use two symbols, $\sqrt{ }$ means 1 and means 10.


Note: the two transcriber defined symbols would be defined either on the special symbols page or in a transcriber's note.

### 11.6 Embellished capital letters

Embellished capital letters are often used to name common sets such as the universal set $E$, the set of real numbers $R$ or integers $I$. These vary in print from book to book but can be represented in braille by the script typeform indicators.

The set of real numbers $\mathfrak{R}$
\%: : : : : : :: : : : : : : : : : : : : : : : :

### 11.7 Greek letters

Greek letters are used heavily in Mathematics. The alphabet is listed below. Refer also to the Rule on Letters and their modifiers.

| : :8: | $\alpha$ Greek alpha | : : : : : : ! : | A capital Greek alpha |
| :---: | :---: | :---: | :---: |
| : $:$ | $\beta$ Greek beta | : : : : : : : | B capital Greek beta |
| : | $\gamma$ Greek gamma | : : : : : : : | $\Gamma$ capital Greek gamma |
| : | $\delta$ Greek delta | : : : : : : | $\Delta$ capital Greek delta |
| ! | $\varepsilon$ Greek epsilon | : : : : : : | E capital Greek epsilon |
| $\bigcirc$ | $\zeta$ Greek zeta |  | Z capital Greek zeta |
| : | $\eta$ Greek eta | : : : : : | H capital Greek eta |
| : $:$ | $\theta$ Greek theta | : : : : : : : | $\Theta$ capital Greek theta |
| ! | 1 Greek iota |  | I capital Greek iota |
| : | $\kappa$ Greek kappa | : $:$ : : : 0 | K capital Greek kappa |
| : | $\lambda$ Greek lambda | : : : : : $:$ | $\Lambda$ capital Greek lambda |
| : 0 | $\mu$ Greek mu | : : : : : : : | M capital Greek mu |
| : $:$ | $\nu$ Greek nu | : : : : : : : : | N capital Greek nu |
| - 0 | $\xi$ Greek xi | : : : : : : : 0 | $\Xi$ capital Greek xi |
| ! | o Greek omicron | : : : : : : : | O capital Greek omicron |
| $\bigcirc$ | $\pi$ Greek pi | : : : : : : : | $\Pi$ capital Greek pi |
| : | $\rho$ Greek rho | : : : : : : : | P capital Greek rho |
| $\bigcirc$ | $\varsigma$ or $\sigma$ Greek sigma |  | $\Sigma$ capital Greek sigma |
| ! : : : : | $\tau$ Greek tau | : : : : : : : | T capital Greek tau |
| $\bigcirc$ | $v$ Greek upsilon | : $: 3:$ | Y capital Greek upsilon |
| $\bigcirc: \%: 0:$ | $\varphi$ Greek phi | : : : : : $:$ | Ф capital Greek phi |
| $\bigcirc$ | $\chi$ Greek chi | : : : : : : : \% | X capital Greek chi |
| : : : : : : | $\psi$ Greek psi | : : : : : : : : | $\Psi$ capital Greek psi |
| ! : : : : : | $\omega$ Greek omega | : : : : : : : | $\Omega$ capital Greek omega |

## 12 Bars and dots etc. over and under

$\because: \vdots$ : bar over previous item
$\because::!$ : $:$ bar under previous item
$\because::$ : $:$ line through previous item (cancellation, "not")
$: \vdots:: \quad$ : simple right-pointing arrow over previous item
$:::::$ : $:!$ simple right-pointing arrow under previous item
$: \vdots::$ : $:$ dot over previous item
$:::$ : : : : dot under previous item
: : : : : : t tilde over previous item
$\because::::!: \quad$ tilde under previous item
$\because:!:!$ hat over previous item
: :: : : : : : hat under previous item
$:::$ : : : : $:$ arc over previous item

### 12.1 The definition of an item

The definition of an item below is the same as that given for superscripts and subscripts in Section 7.1.

As in Section 7, an item is defined as any of the following groupings:

1. An entire number, i.e. the initiating numeric symbol and all succeeding symbols within the numeric mode thus established (which would include any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
2. An entire general fraction, enclosed in fraction indicators (see Section 6).
3. An entire radical expression, enclosed in radical indicators (see Section 8).
4. An arrow (see Section 13).
5. An arbitrary shape (see Section 14).
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces (see Section 5).
7. Any expression enclosed in the braille grouping indicators
8. If none of the foregoing apply, the item is simply the next individual symbol.

## Examples:

$\bar{x}=\frac{10+11+12}{3}$ where $\bar{x}$ is the arithmetic mean.
( $x$ bar equals $10+11+12$ all over 3 where $x$ bar is the arithmetic mean.)


Note: the second occurence of $\bar{x}$ needs a grade 1 symbol indicator because it is within a grade 2 passage.

```
\(x+y \quad(x+y\) all with a bar under \()\)
: : : : : : : : : : : : : : : : : : : : :
\(\neq \quad\) (not-equals)
: :: : : : :
```

0.3 ( 0.3 with a dot over the 3 - the recurring decimal 0.33333 $\ldots$ )

```
:: :: :%:.: : : : : : : : : :
```

Note: braille grouping signs are needed here otherwise the dot would refer to the entire number.
$0.5612 \dot{3}$ (The recurring decimal $0.56123123123 \ldots$ )


Because the digits 123 are being implicitly grouped in print this can also be written as:


```
derivatives }\dot{x}\mathrm{ and }\ddot{x}\quad\mathrm{ (derivatives }x\mathrm{ dot and }x\mathrm{ double dot)
```


Angle $A \hat{B} C \quad$ (Angle $A B C$ with a hat or caret over the $B$ )


### 12.2 Two indicators applied to the same item

If two indicators apply to the same item, then braille grouping symbols must be used to show which applies first.

```
X
```



```
\mp@subsup{x}{}{y}}\quad\mathrm{ (x to the y with a bar over the whole expression)
```



## 13 Arrows

### 13.1 Simple arrows

| :: : |  | arrow indicator |
| :---: | :---: | :---: |
| : : : : : |  | bold arrow indicator |
|  | $\rightarrow$ | simple right pointing arrow (east) |
| : $: \%$ : : $:$ | $\leftarrow$ | simple left pointing arrow (west) |
| : : : : : | $\uparrow$ | simple up pointing arrow (north) |
| : : : : : | $\downarrow$ | simple down pointing arrow (south) |
| : : : : : $:$ | 7 | simple up and right pointing arrow (northeast) |
| :: : : : | $\rangle$ | simple down and right pointing arrow (southeast) |
| :: : : : : | $\checkmark$ | simple up and left pointing arrow (northwest) |
|  | $\checkmark$ | simple down and left pointing arrow (southwest) |

A simple arrow has a standard barbed tip at one end (like a v on its side, pointing away from the shaft). The shaft is straight and its length and thickness are not significant. These arrows are represented by an opening arrow indicator and the appropriate closing arrow indicator. Notice that all these terminating symbols have three dots, arranged in a consistent pattern that best describes the direction.

Note that unless you are already in grade 1 mode, the arrow indicator will need a grade 1 symbol indicator. The bold arrow indicator will not need one, as this two cell symbol does not have a grade 2 meaning. Both arrow indicators set arrow mode so no further grade 1 indicators will be needed.

Arrows are signs of comparison so should usually be spaced. An exception is when they are written below the limit function (see Section 9.6).

Do not use arrow indicators when a simple right pointing arrow is the only modifier above or below an item. See "arrow over previous item" and "arrow below previous item" in Section 12.
$\mathrm{n} \rightarrow 0 \quad$ ( n right arrow $0-\mathrm{n}$ tends to zero)

input $\rightarrow$ process $\rightarrow$ output
(input "right arrow" process "bold right arrow" output)

### 13.2 Arrows with unusual shafts and a standard barbed tip

## Shaft symbols:

$\because: \vdots \quad$ short single straight line
$\because: \%$ medium single straight line
: : : : : : : 0 long single straight line
: :: :: double, short
: : : : : : : : dotted, long
$\because: \quad$ : $: \quad$ curved or bent to the left (anticlockwise in line of direction)
$\vdots:!\quad$ : curved or bent to the right (clockwise in line of direction)
$\because: \%$ sharp turn to the right (in line of direction)
$\because: \%$ sharp turn to the left (in line of direction)

All shaft symbols can be elongated by repetition. The shaft symbols are placed between the opening and closing arrow indicators. Arrow length only needs to be indicated in braille when in print arrows of different lengths have different meanings.

These examples still have standard barbed tips.

```
(double shafted medium length right pointing arrow)
    :%:
\Gamma (medium arrow pointing up with a sharp turn to the right)
    :0:%:0:%:0
    \curvearrowright (medium length right pointing arrow bending clockwise)
:!
```

```
\leftarrow - - - - (long, broken left pointing arrow)
::!:!:!:!: !: !
```



```
(short, medium and long down pointing simple arrows)
```



### 13.3 Arrows with unusual tips

Barb symbols:
$\vdots: \%$ regular barb, full, in line of direction
$\because: \%$ regular barb, full, counter to line of direction
: : : : : : : regular barb, upper half, counter
: :: : : :: regular barb, lower half, counter
: : : : : : regular barb, upper half, in line
$\vdots:: \vdots$ regular barb, lower half, in line
$\because: \%$ curved, full, counter
$\vdots:: \quad$ curved, full, in line
$\because: \vdots:!$ curved, upper half, counter
$\because:: \vdots: \% \quad$ curved, lower half, counter
$\vdots::!: \quad$ curved, upper half, in line
$\vdots:: \vdots: \quad$ curved, lower half, in line
$\because: \%$ : $: \quad$ straight, full, (directionless)
$: \vdots::$ : straight, upper half, (directionless)
$\vdots::$ : : : : straight, lower half, (directionless)
If an arrow has unusual tips, decide which is the head before you choose the direction of your closing indicator. The complete rules for deciding arrow direction are:

1. If there are directional tips, and all lead in the same direction, the head is the end that lies in that direction.
2. If there are no directional tips, but one end has a tip and the other does not, the end with the tip is the head.
3. In all other cases, the head of the arrow is deemed to be the end at the right, or in the case of strictly vertical arrows, at the top.

The tip(s) and shaft segment(s) are transcribed between the opening and closing indicators. These items are expressed in logical order, that is starting with the arrow tail and progressing towards the head, even if that runs counter to the physical order (as in the case of a left pointing arrow). Certain elements are omitted, corresponding to these reader rules:

1. If no tip is transcribed, it is understood that an ordinary full barbed tip occurs at the arrow head, and there is no other tip.
2. If no shaft is transcribed, it is understood that the shaft is a straight line of medium length. In this case, if no tip is transcribed, rule (1) also applies; if one tip is transcribed, it is at the head; if two tips are transcribed, the first is at the tail and the second at the head.
$\rightarrow \quad$ (otherwise ordinary right arrow, with curved head)
: : : : : :
$\leftrightarrow \quad$ (common horizontal bidirectional arrow)
: : : : : : : : :
$\downarrow$
(horizontal bidirectional arrow, tilted from lower left to upper right)
:\%:\%:
$\rightarrow \quad$ (otherwise ordinary right arrow, with tail and head tips)

## : : : : : : : :

$\rightarrow \quad$ (otherwise ordinary right arrow, with two head tips)

$\longleftarrow$
(otherwise ordinary left arrow, with tail and head tips)

```
!:
```

$\leftarrow \quad$ (otherwise ordinary left arrow, with two head tips)
: : : : : : : : : : : : : :
$\mapsto \quad$ (right arrow with a straight tail tip and a normal head tip)
: : : : : : : : : : :
$\uparrow \downarrow$
(bold arrow up, followed by ordinary arrow down)
: : : : : : : : :
$\rightleftarrows \quad$ (common right arrow over common left arrow see Section 14.3c regarding vertical juxtaposition)
: : : : : : : : : : : :
$\leftrightharpoons \quad$ (half-barbed left arrow over half-barbed right arrow reversible chemical reaction)


See Section 16 for equilibrium arrows that occur in Chemistry.

## 14 Shape Symbols and Composite Symbols

## Listing of shape indicators:

$:: \%$ shape indicator
$:::$ : : : : filled (solid) shape indicator
$\because::$ : : : $:$ shaded shape indicator
$\because:!$ transcriber-assigned shape indicator
$::$ : : : : : transcriber-assigned filled (solid) shape indicator
$::::!:!$ transcriber-assigned shaded shape indicator
$\vdots: \%$ : shape terminator

## Listing of specific shapes:

: : : : : : : : :
: : : : : : : : : : square
$:: \%:!:!: \quad$ regular pentagon
: : : : : : : : :
: :: : : : : : : : regular heptagon
: : : : : : : : regular octagon (etc. for all regular polygons)
$\because: \%: \%$ circle
: : : : : : : : : : : parallelogram

## Composite Symbols:

$:: \%$ superposition indicator
: :: : :: horizontal juxtaposition indicator
$\because: \%$ vertical juxtaposition indicator
: : : : : physical enclosure indicator

### 14.1 Use of the shape termination indicator

14.1.1 If a shape is followed by a space then no termination symbol is needed.
$\triangle A B C \quad$ (triangle symbol space $A B C$ )

14.1.2 If the shape symbol is followed by punctuation, or unspaced from a following symbol, then the shape terminator must be used.
$\triangle A B C$


What is the next shape?


Note that unless you are already in grade 1 mode, a grade 1 symbol indicator will be needed before the shape indicator. This does not however apply to the shaded and filled shape indicators because these two cell symbols do not have a grade 2 meaning. All the initial shape indicators initiate shape mode so no further grade 1 indicators will be needed.

### 14.2 Transcriber defined shapes

The description within transcriber defined shapes should be a short series of initials or a single grade 1 word. They should not be used if the print symbol is already covered elsewhere in the code. The definitions of all shape symbols should be available to the reader in either a transcriber's note or on a special symbols page.

For example, a smiling face $)$ used as an icon throughout a book could be defined as

rather than

which is too long and requires braille grouping symbols to stop the space terminating the shape.

### 14.3 Combined shapes

If two print symbols have been combined to form a new previously undefined symbol, then it must be decided whether the second symbol is enclosed, superimposed, combined on the right or combined below. Each of the four composite symbol indicators signals a combining of the item just prior with the item immediately following it, where "item" is as defined in Section 7.

Each composite symbol indicator will need a grade 1 symbol indicator unless the whole expression is already in grade 1 mode.

### 14.3.1 Physical Enclosure, : : : :

$\oplus \quad$ (circle enclosing a plus sign)


In the example below, the circle enclosing a plus sign is being used as an operation sign. In the first version the operation sign is unspaced so a termination sign is needed but no grade indicators are needed because the number signs initiate numeric mode. In the second version the operation sign is spaced for clarity so no termination sign is needed but the shape symbols do need grade 1 indicators.
$2 \oplus 3=3 \oplus 2 \quad(2$ "circled plus" $3=3$ "circled plus" 2$)$

$2 \oplus 3=3 \oplus 2 \quad(2$ "circled plus" $3=3$ "circled plus" 2)


Note: See Section 3.5 if numbers are enclosed in a square to represent a calculator key.

### 14.3.2 Superposition, : : :

Note that this structure should not be used for negation. See "line through previous item" in Section 12.

R ( R with superimposed x - prescription symbol)

```
!::%:%:
```

$\oint$
(Integral sign with a small circle superimposed half way up closed integral defined in Section 11.1)
: : :
§
(integral sign with a small square superimposed half way up the termination could be omitted if there was a following space)

```
: :0:: :: :%:: :: ::
```


### 14.3.3 Vertical Juxtaposition, :: ::

The upper symbol should be given first, followed by the vertical juxtaposition indicator, then the lower symbol.

This structure should not be used for bars, arrows, dots, tildes or hats over or under other symbols (see Section 12). Neither should it be used for superscripts or subscripts written directly over or under (see Section 7).
$\xlongequal{\circ}$ (a hollow dot with an equal sign underneath)
: : : : : : : : :
$\binom{n}{r} \quad$ binomial coefficient (refer Section 11.5)


### 14.3.4 Horizontal Juxtaposition, :: ::

"Horizontal juxtaposition" is to be invoked only when two symbols are written in close proximity and it is clear from the usage that a new single symbol, distinct from the elementary symbols considered in sequence, is intended. Otherwise, symbols written one after the other should simply be brailled accordingly.

## 15 Matrices and vectors

### 15.1 Enlarged grouping symbols

When enlarged brackets are used in print for vectors, matrices, systems of equations, function definitions etc., the appropriate enlarged grouping symbols should be used in braille. These are the usual grouping symbols preceded by a dot 6 . See the full list in Section 5 . These should be placed directly under each other. Blank lines before and after such arrangements may be needed for clarity.

### 15.2 Matrices

The columns should be left adjusted except for minus signs which should be brailled to stand out. One column of blank cells should be left between columns. Material outside the matrix, such as signs of operation and comparison, should be placed on the top line, even if they are centered in print.

$$
\begin{aligned}
& I=\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) .
\end{aligned}
$$

$$
\begin{aligned}
& {\left[\begin{array}{lll}
1 & 2 & 3 \\
4 & 5 & 6
\end{array}\right]\left[\begin{array}{cc}
1 & 2 \\
-3 & 4 \\
5 & -6
\end{array}\right]} \\
& \left(\begin{array}{cc}
a & -b \\
-c & d
\end{array}\right)
\end{aligned}
$$

### 15.3 Determinants

These have the same structure as matrices but are normally enclosed in print with enlarged vertical bars.
$|P|=\left|\begin{array}{ll}a & b \\ c & d\end{array}\right|=a d-b c$

$$
\begin{aligned}
& \text { 4:\% : : }
\end{aligned}
$$

### 15.4 Omission dots

The placement of dots used to indicate the omission of one or more rows or columns can follow the print.

$$
\left|\begin{array}{cccc}
a_{11} & a_{12} & \ldots & a_{1 n} \\
a_{21} & a_{22} & \ldots & a_{2 n} \\
\cdot & \cdot & \ldots & \cdot \\
a_{m 1} & a_{m 2} & \ldots & a_{m n}
\end{array}\right|
$$



### 15.5 Dealing with wide matrices

If a matrix or determinant is too wide for the braille page, runovers within entries may be necessary. If there is not room to indent these runovers, they can be blocked and a blank line left between rows.
$\left(\begin{array}{lll}a_{1} x_{1}+b_{1} x_{2}+c_{1} x_{3} & a_{1} y_{1}+b_{1} y_{2}+c_{1} y_{3} & a_{1} z_{1}+b_{1} z_{2}+c_{1} z_{3} \\ a_{2} x_{1}+b_{2} x_{2}+c_{2} x_{3} & a_{2} y_{1}+b_{2} y_{2}+c_{2} y_{3} & a_{2} z_{1}+b_{2} z_{2}+c_{2} z_{3} \\ a_{3} x_{1}+b_{3} x_{2}+c_{3} x_{3} & a_{3} y_{1}+b_{3} y_{2}+c_{3} y_{3} & a_{3} z_{1}+b_{3} z_{2}+c_{3} z_{3}\end{array}\right)$

© : : : : :
Another approach is to complete the first column without overruns and then to place the next column below this, indented two cells.

```
: :: :%: :: :: ::
```








```
    !
    !
```


: : : : : : :
Notice that in the first example the structure of the matrix is clearer but in the second example the individual entries are easier to read. Notice also the different placement of the enlarged grouping signs in the two examples.

A transcriber's note should be included in either case to explain the placement of each entry.

### 15.6 Vectors

Letters representing vectors are often printed in bold font and may have arrows or bars above or below. Boldface only needs to be shown in braille if it is the only method used. For arrows and bars above and below see Section 12.

If the vector $\binom{2}{-1}$ was called $p$ and went from point $A$ to point $B$ here are some of the most likely forms:

$$
\underline{p}=\binom{2}{-1}
$$

( $p$ with a bar under $=$ enlarged round brackets, 2 at the top and -1 at the bottom)

$\bar{p} \quad$ ( $p$ with a bar over)
: : : : :
p (bold p)
:: : : :
$\overrightarrow{A B} \quad$ (AB with an arrow over)
: :: : : : : : : : : : : : : : : :

AB (AB with a bar under)
: :!:! : : : ! : : : : : : : :

### 15.7 Grouping of equations

Opening enlarged curly braces are often used to group equations. Print spacing should be followed where possible.

Solve:

```
\(\{x+2 y=7\)
\(2 x-y=-4\)
! : : : : : : : : : : : : : :
    \(\because: \vdots: \quad\) : : : : : : : : : : : : : :
```


$f(x)=\left\{\begin{array}{c}0 \text { if } x<0 \\ x^{2} \text { if } x>0\end{array}\right.$



## 16 Chemistry



### 16.1 Chemical names

cyclohexane

```
:
```

2,2-Dimethylpropane

```
:0
```

1-Hydroxyl,2-nitrobenzene


Copper(II) Sulphate

```
*:%:%:%:%%%%%%%% % %%:%:%%:%%
```

Copperll Sulphate


### 16.2 Chemical formulae

Use of capital indicators and terminators: The general UEB principles on the choice of single letter, word or passage mode apply; in particular, a capital terminator should not be used within a two-letter chemical element symbol. Using single capital indicators for chemical formulae provides a uniform appearance to the braille; nevertheless, there may be a clear advantage in using capital passage mode in some cases. Letters representing chemical elements should never be contracted.

## 0

Fe
NO
NaCl
$\mathrm{Al}(\mathrm{O}) \mathrm{Cl}$
$\mathrm{O}_{2}$



An example of using capital passage mode:
$\mathrm{R} \cdot \mathrm{CH}(\mathrm{OH}) \cdot \mathrm{CH}_{2} \cdot \mathrm{CH}_{2} \cdot \mathrm{CO}_{2} \mathrm{H}$

Note that Roman numerals are capitalised as a single group:

$$
\mathrm{Fe}^{\text {II'Cl }} \mathrm{Cl}_{3} \quad \text { : }
$$

### 16.3 Atomic mass numbers

These are often shown with left-hand superscript and subscript numbers:

```
2 3 8
```



### 16.4 Electronic configuration

Such notation can involve left-hand superscripts, and a space or half-space in print needs to be retained in the braille to make the attachment clear.

Ar $1 s^{2} 2 s^{2} 2 p^{6} 3 s^{2} 3 p^{6}$

```
:
```

${ }^{1} S_{0}$

### 16.5 Chemical Equations

$2 \mathrm{NaOH}+\mathrm{H}_{2} \mathrm{SO}_{4} \rightarrow \mathrm{Na}_{2} \mathrm{SO}_{4}+2 \mathrm{H}_{2} \mathrm{O}$




```
or
: : : : : : : : :
```



```
: : : : : :
```

$\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~s})=2 \mathrm{HI}(\mathrm{g})$

: : : : : : : : :
$\mathrm{HNCO}+\mathrm{ROH} \rightarrow \mathrm{NH}_{2} \cdot \mathrm{CO} \cdot \mathrm{OR} \rightarrow \mathrm{NH}_{2} \mathrm{CO} \cdot \mathrm{NH} \cdot \mathrm{CO}_{2} \mathrm{R}$


$\mathrm{Pb}^{++}+2 \mathrm{e} \rightleftharpoons \mathrm{Pb}$


### 16.6 Electrons

Electrons shown as dots, crosses or circles attached to element symbols may be represented using the corresponding UEB signs for single, double, triple dots, etc. However, more complex cases (e.g. including graphic lines) are better dealt with using ordinary diagram methods, rather than trying to code them purely in braille.

Cl .


### 16.7 Structural Formulae

The general UEB method for line drawing may be used for representing structural formulae. Refer to the Line Drawing Rule for common line types defined in UEB, others may be assigned as user-defined symbols. In-line bond signs can be used in conjunction with the line drawing symbols.

Some examples are shown below. For more complex cases it is recommended that the structure be drawn out graphically rather than using transcriber defined symbols.


H

: : : : : :
: : : : : : : : : : : : : : : :



: : : : : : : : : : : : : :


: : : : : : : : : : : : :

6: : : : : : : : :


## Linear Method

A method is available in which structural formulae are represented in a compact linear form in braille. However, as yet this has not been documented as a UEB technique.

## 17 Computer Notation

$\vdots: \vdots: \quad$ (on a line by itself) cursor indicator
: :: : : visible space
: : : (at end of line) continuation indicator
$\vdots: \vdots$ (at end of line) continuation indicator with space
: :: : : :: nondirectional double quote (ASCII double quote)

### 17.1 Definition of computer notation

Computer notation is any text written in a formal syntax that is designed to allow computers to utilize the text directly for technical purposes related to the computer itself. Examples include computer programs written in procedural languages such as Java, C++, COBOL, and various "assembly" languages, nonprocedural scripting and markup languages such as XHTML, and data files prepared to meet the input requirements of specific programs. Even some short items such as email addresses, Web site URLs, and file names qualify as computer notation because of their technical purpose. Other examples might occur within an instruction manual for technical equipment, or within a statistics textbook that gives an example of a spreadsheet formula.
"Displayed" computer notation is presented in one or more lines separate from the surrounding literary text; "inline" computer notation is presented within ordinary literary text -- for example, an email address mentioned within a sentence.

Historically, separate codes have been used for such computer notation. In UEB this is no longer the case, as each print symbol is represented by the same braille symbol, regardless of whether it appears in a literary, mathematical or computer context. Refer to the list of miscellaneous symbols at the start of Section 11 for symbols on the computer keyboard such as the <br>(backslash), ~ (tilde), @ (at sign), \# (cross hatch), \& (ampersand), * (asterisk), _ (underscore) and | (vertical bar).

### 17.2 Line arrangement and spacing within computer notation

The significance of line breaks within displayed computer notation depends upon the specific formal syntax involved. Unless the transcriber is certain that such line breaks are not significant, it should be assumed that they are, and the line-by-line arrangement preserved in braille. When the limited length of the braille line makes a break necessary in braille that does not correspond to a break in print, a continuation indicator should be inserted and the line continued on the next line. The specific continuation indicator used depends on whether the break in effect replaces a space in the original line (which is preferred, if practical, and in which case use dots 5,5 ), or is simply introduced at an
arbitrary point (in which case use dot 5). In the latter case, when practical, it is preferable not to introduce a break between letters.

The significance of spaces within computer notation also depends upon the specific formal syntax involved. Exact spacing, that is a certain number of spaces, may sometimes be significant, as in "strings" -- although usually, only the presence or absence of space is significant. If the print presentation makes the number of spaces practical to count, as for example by using "visible spaces" (see below) or a monospaced font, and the transcriber is not certain that exact spacing is not significant, then it should be assumed that exact spacing is significant. Otherwise, spaces should be used where they are in print, but not necessarily the exact same number. As a particular application of the latter case, when the left margin shows a pattern of indentation, such indentation should be replicated in braille, but generally using multiples of two cells instead of the five or ten positions often used in print.

In print computer notation, a "visible space" can be represented by any of several adhoc characters, such as:
b U+0180 Latin small letter b with stroke (or "b" plus combining stroke or solidus)
$\llcorner\quad U+23 B 5$ bottom square bracket
$\Delta \quad$ U+0394 Greek capital letter delta
$\Delta \quad$ U+2206 Increment
$\Delta \quad \mathrm{U}+25 \mathrm{~B} 3$ White up-pointing triangle

When such "visible spaces" are used in print, dots 346 should be used in braille for each such "visible space" (preceded by grade 1 indicators if necessary). Likewise, if per the guidelines above it is determined that a sequence of three or more ordinary spaces in print is to be treated as significant, then each significant space in the sequence, other than the first and the last, should be indicated by dots 346 (with grade 1 indicators if necessary).

## Examples:

1. A "command prompt" showing the cursor immediately following (in grade 1 context):
```
C:\>
    #
```

    ::
    2a. A "string" containing explicit "visible spaces" (grade 2 context):
PRINT "Name $\Delta \Delta \Delta \Delta T e l . \Delta N o . "$

2b. The same example in grade 1 context:
3. A "string" containing implied significant spaces (grade 1 context):

PRINT "Name Tel. No."

4. Runovers made necessary because of a 32 -cell line limit, within computer notation (grade 2 context):

You can find the story at
http://www.ridbc.org.au/news/stories_children.asp

5. A runover at a space (grade 2 context):

The program is stored in directory c:\program files\abc corporation\utility programs


### 17.3 Grade of braille in computer notation

A displayed computer program or program fragment should normally be transcribed in grade 1 braille; nearby excerpts from a program that is displayed in grade 1 should preferably also be in grade 1, for consistency. Other expressions, such as email addresses, web sites, URLs, filenames, and computer expressions not displayed on separate lines, should normally be transcribed in grade 2 braille.

## Example:

Now that you've seen the "Hello World!" application, you might be wondering how it works. Here again is its code:

```
/**
    * The HelloWorldApp class implements an application that
    * simply prints "Hello World!" to standard output.
    */
class HelloWorldApp {
    public static void main(String[] args) {
                System.out.println("Hello World!"); // Display
the string.
    }
}
```

The call to function system. out. println, passing it the constant string "Hello World!", does the essential work -- but the other statements and comments also play a role ...

## Notes:

The indentation pattern on the left of the page has been followed in braille. The "continuation with space" indicators have been used whenever a print line needed to be broken at a space in braille. Overruns following a continuation indicator are indented in the braille version below, but it is also acceptable to start each overrun in cell 1 . Notice that the non directional double quotation marks around "Hello World" are treated as standard quotes in the introductory paragraph but have been brailled exactly as written when they appear in the program itself and in the extract. In the final paragraph, grade 1 indicators were used for the two extracts so it was not felt necessary to also show the change of font.


